

Possibility of self-similar pulse evolution in a Ti:sapphire laser

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Abstract: A theoretical investigation of the possibility of achieving self-similar pulse propagation in a solid-state laser is presented. Limited group-velocity dispersion hinders true self-similar pulse evolution, but an intermediate regime that exhibits some of the characteristic features (and offers some of the benefits) of self-similar propagation can be reached. This regime of operation offers the potential to increase the pulse energy by at least an order of magnitude compared to energies obtained in the usual operation of Kerr-lens mode-locked lasers with anomalous dispersion. Ti:sapphire lasers that generate pulse energies as high as one microjoule and peak powers of ~ 100 MW should be possible based on this mode of operation.

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Femtosecond solid-state lasers, particularly those based on the Ti:sapphire gain medium, have revolutionized the field of ultrafast science in the past decade following the discovery of the Kerr-lens mode-locking [1]. The simplicity, stability, and excellent performance of these lasers have led to their widespread use. Nevertheless, researchers continue to investigate approaches to improving the performance of mode-locked solid-state lasers. The most visible results of this effort are perhaps in the area of pulse duration, which has been reduced from ~ 100 fs to 5 fs, which corresponds to approximately two cycles of the electromagnetic field [2, 3, 4, 5]. Most of this improvement has resulted from greater control of the cavity dispersion. Interestingly, the lasers that produce the shortest pulses operate in a mode that is analogous to a stretched-pulse fiber laser [6]: the pulse evolves like a dispersion-managed soliton, breathing as it propagates around the cavity [7]. In this mode of operation, the main role of Kerr lensing is to provide stability against noise buildup. The main pulse-shaping dynamics, which are dominated by dispersion and Kerr nonlinearity [8], can be understood with one-dimensional computer simulations [7].

A number of techniques for increasing the pulse energy that can be obtained from solid-state lasers have also been reported, including cavity-dumping [9] and the construction of low-repetition-rate lasers by the inclusion of a multi-pass cavity in the resonator [10, 11]. In these lasers the pulse duration varies little throughout the cavity, and the operation can be understood in terms of the theory of passive mode-locking based on the assumption of small changes per pass [12, 13]. One of the routes to higher pulse energy is to operate the laser with normal net cavity group-velocity dispersion (GVD) [14]. This produces highly-chirped output pulses, which can be dechirped to some degree outside the laser. In this mode of operation, the pulse duration is nearly constant within the cavity. As the pulse energy is pushed to higher values, excessive nonlinearity becomes an issue and is emerging as a limitation to the performance of these lasers.

Excessive nonlinearity is an over-riding concern in the design of high-energy short-pulse fiber lasers. Small transverse modes in fibers produce high intensities even with moderate pulse energies. The resulting nonlinear phase shifts can cause wave-breaking, which is typically manifested experimentally as instability, or as the formation of secondary pulses in the cavity. Stretched-pulse fiber lasers relieve the limitation on pulse energy: the breathing solutions can contain energies an order of magnitude larger than those of static solitons. Recently, a new regime of mode-locking has been demonstrated in fiber lasers: self-similar pulse evolution allows wave-breaking-free operation to be achieved with pulse energies much greater than can

be attained by stretched-pulse lasers [15, 16]. Theoretically, such “similariton” pulses reach energies up to two orders of magnitude larger than those of stretched-pulse lasers with increasing normal cavity dispersion. The first experimental demonstrations of self-similar operation of fiber lasers produced 3 times larger pulse energy, and 5 times larger peak power, than the corresponding maxima obtained with stretched-pulse lasers.

Considering the unabated interest in producing higher-energy pulses with mode-locked lasers, it is natural to ask whether the concept of self-similar (or wave-breaking-free) pulses can be applied to solid-state lasers. In this paper we outline the issues that must be addressed to operate a solid-state laser in the similariton regime. Numerical simulations of a solid-state laser with realistic parameters will be presented. These show that the relatively small dispersion of the segments of the solid-state laser precludes operation with large stretching ratios, so the full benefits of self-similar evolution cannot be exploited. With stretching ratios of only ~ 2 , the characteristic features of self-similar evolution begin to appear, and ratios up to 10 appear to be feasible. Significantly, pulse energies substantially larger than those obtained in the usual operation with net anomalous dispersion should be possible. Thus, we expect that there may be practical advantages to self-similar operation of solid-state lasers.

We briefly review the main issues and properties of a similariton laser, to provide context for the discussion of a solid-state version. Self-similar pulses are asymptotic solutions of the nonlinear Schrödinger equation [17, 18], and their monotonic evolution must be reconciled with the periodic boundary condition of a laser cavity. An anomalous-GVD segment is required in a similariton laser, and this should have negligible nonlinearity, so that there is no tendency to form a soliton. Finally, self-similar propagation of intense pulses is disrupted if the pulse encounters a strong limitation to its spectral bandwidth [19]. We can think of both fiber and solid-state lasers most simply as consisting of segments of normal and anomalous GVD, with a positive (self-focusing) nonlinearity in the normal-GVD segment. Self-similar pulse propagation occurs when the net cavity GVD is large and normal. In the self-similar regime, the pulse is (positively) chirped throughout the cavity. The pulse duration and chirp increase monotonically in the normal-GVD segment, and decrease monotonically in the anomalous-GVD segment. The pulse never reaches the transform-limited duration in the cavity, but it can be dechirped to the transform-limit external to the cavity. In the frequency domain, bandwidth is generated in the normal-GVD (and nonlinear) segment, and then cut off in the gain medium (and possibly in an effective saturable absorber as well).

Self-similar operation of a solid-state laser would extend the previously-observed operation with normal GVD [14, 10], where the pulse duration is nearly static, to stretching ratios larger than one. It is worth mentioning that there is no analytical theory of mode-locking under these conditions, which include the possibility of large changes in the pulse parameters with propagation. In both the solid-state and fiber lasers, anomalous dispersion can be provided with negligible nonlinearity. In the similariton fiber laser, the potentially-adverse effects of the gain bandwidth on the pulse are avoided by placing a very short gain segment after a long stretch of undoped fiber that constitutes most of the laser. The huge gain bandwidth of Ti:sapphire can be expected to ameliorate this problem, at least for pulse durations longer than ~ 10 fs (all pulse durations are expressed as full-widths at half-maxima, FWHM). The most difficult condition to satisfy is a large stretching ratio, which requires substantial normal GVD. A fiber laser comprises 10-100 characteristic dispersion lengths, while a solid-state gain crystal is comparable to the dispersion length only for ~ 10 -fs pulses. An equivalent statement is that the solid-state laser has normal dispersion ~ 500 times smaller than the fiber laser, so the pulse bandwidth must be ~ 20 times larger to achieve a similar stretching ratio (and therefore increase in energy). Only a two-cycle laser can approximate the intra-cavity breathing of a fiber laser.

Numerical simulations provide excellent qualitative, and semi-quantitative, predictions of the

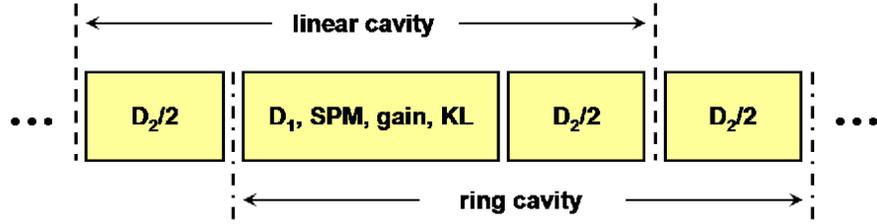


Fig. 1. Schematic of a laser cavity showing the main effects on the dynamics of the pulse generation. The imposition of the periodicity distinguishes between linear and ring cavity configurations. D_1 : normal GVD, D_2 : anomalous GVD, SPM: self-phase modulation, KL: Kerr lens.

behavior and performance of fiber similariton lasers [15, 16]. The model used here is modified, namely a distributed saturable absorber (SA) is utilized in the gain section. Stable solutions that evolve from noise were found for reasonable ranges around each set of parameters discussed below. The laser is modeled as having two segments: a positive-GVD (i.e., normal dispersion) segment with positive self-phase modulation (SPM), and gain, followed by a linear dispersive delay (Fig. 1). Propagation is modeled with an extended nonlinear Schrödinger equation

$$\frac{\partial}{\partial z} a(z,t) = -i\frac{1}{2}\beta(z) \frac{\partial}{\partial t^2} a(z,t) + i\gamma|a(z,t)|^2 - q(a(z,t))a(z,t) + \frac{g_0}{1 + E_{\text{total}}/E_{\text{sat}}} \frac{1}{\Delta\omega_{\text{gain}}} \frac{\partial}{\partial t^2} a(z,t), \quad (1)$$

where $a = a(z,t)$ is the envelope of the field, $\beta(z)$ is the GVD parameter which is step-wise constant, and γ is the SPM coefficient which is non-zero only for the positive-GVD segment. The gain medium has a long relaxation time in comparison to one cavity round trip time, is saturated by a series of successive pulses, and has parabolic frequency dependence with a bandwidth of $\Delta\omega_{\text{gain}}$. E_{total} is the total intra-cavity energy, g_0 is the small signal gain, E_{sat} is the gain saturation energy. The Kerr-lens effect is modeled as a fast SA of the form $q(a(z,t)) = q_0/(1 + |a(z,t)|^2/q_{\text{SA}})$, where q_{SA} is the saturation power and $q_0 = 0$ outside the Kerr medium.

Parameters typical of Ti:sapphire lasers (Table 1) were chosen for the simulations. The output coupler is chosen to be 10%. Other linear losses are ignored. As a reference, operation of a Ti:sapphire laser with small anomalous GVD was simulated first. With net GVD of -3.0 fs^2 , nearly transform-limited 10-fs pulses with intra-cavity energy of 55 nJ are generated (Fig. 2). At this pulse energy, the peak power within the Ti:sapphire crystal is 5 MW, which is low enough to avoid instabilities and material damage due to self-focusing in the short gain crystals that are typically employed. Higher-order dispersion is neglected, to better isolate the dominant dynamical effects (GVD, SPM, and filtering by the gain bandwidth) under scrutiny. Precise compensation of higher-order dispersion with the use of chirped mirrors is well-established. Furthermore, the demonstration of self-similar operation in a fiber laser with strong higher-order dispersion arising from diffraction gratings suggests that this mode of operation is relatively robust against higher-order dispersion.

Starting from this operating point, the cavity dispersion was increased to net normal values, and solutions that exhibit the self-similar evolution were sought. With the cavity dispersion set to $+15.0 \text{ fs}^2$ the output pulse shown in Fig. 3 is obtained. The pulse energy is increased by ~ 7 times compared to the value that produced the reference 10-fs pulses, along with the saturation power to prevent pulse break-up. Nevertheless, the peak power attained within the Ti:sapphire

Table 1. Set of input parameters corresponding to the numerical simulations for three different configurations A, B, and C. Parameters marked with (*) are outputs, determined by running the simulations. In practice, E_{pulse} is gradually increased from a low value until $I_{\text{peak}} \sim 5$ MW is obtained.

parameter	laser A	laser B	laser C
L , Ti:sapphire crystal length	2.0 mm	2.25 mm	3.0 mm
D_1 , total positive GVD	+120 fs ²	+ 135 fs ²	+180 fs ²
D_2 , total anomalous GVD	-123 fs ²	-120 fs ²	-150 fs ²
$D_{\text{total}} = D_1 - D_2$, total GVD	-3 fs ²	+15 fs ²	+30 fs ²
γ (for normal-GVD segment)	1 (MW.cm) ⁻¹	1 (MW.cm) ⁻¹	1 (MW.cm) ⁻¹
$\Delta\omega_{\text{gain}}$, gain bandwidth	100 THz	100 THz	100 THz
q_0 , modulation depth	0.01/mm	0.01/mm	0.01/mm
q_{SA} , saturation power	0.5 MW	3.5 MW	6 MW
S , intracavity breathing(*)	2.5	2.6	2.0
I_{peak} , peak intra-cavity power(*)	4.5 MW	5 MW	6 MW
t_{min} , min. intra-cavity pulse width(*)	10 fs	47 fs	109 fs
t_{dechirp} , dechirped pulse duration(*)	10 fs	8.5 fs	9 fs
E_{pulse} , intracavity pulse energy(*)	55 nJ	360 nJ	1.0 μJ

crystal remains similar to the value for the negative-GVD results, despite the large increase in pulse energy. The power spectrum exhibits the shape that is characteristic of self-similar pulses: parabolic near the peak, with a transition to a steep decay. The pulse duration monotonically increases throughout the Ti:sapphire crystal (the segment of the cavity with normal dispersion), in agreement with observed behavior in fiber lasers supporting self-similar pulse evolution and in contrast to stretched-pulse lasers (where the pulse duration decreases, attains a minimum, then increases). The pulse shaping within one round trip is illustrated in Fig. 4. The pulse duration varies from ~ 47 fs to ~ 122 fs in the cavity, so the breathing ratio is only 2.6. However, the chirp is large (and positive) everywhere in the cavity: the pulse duration varies from 5.5 to 14 times the transform-limit. Outside the cavity, the pulse can be dechirped to ~ 9 fs (with -23 fs² of dispersion for pulses extracted at the point where they have largest positive chirp). The dechirped pulse duration is comparable to the negative-GVD case, but the spectral bandwidth is larger. The reduced pulse quality is a consequence of the fact that the chirp is not as linear across the pulse as it is in a similariton fiber laser.

The simulations demonstrate that some of the features of self-similar pulse evolution are obtained under these conditions: the characteristic spectral shape is observed along with a reasonably linear chirp and increased pulse energy. Intuitively, the stretching stabilizes a pulse of higher energy than could be tolerated in static evolution. However, the pulse-shaping is partially disrupted by gain filtering, which produces a temporal pulse shape that is not fully parabolic. The deviation from parabolic pulse shape in turn is responsible for the nonlinear chirp.

It is important to contrast this approximately self-similar mode of operation with the “standard” normal-GVD operation of a Ti:sapphire laser [14], which is understood on the basis of the analytical model developed in [12]. Because the analytical model assumes a static pulse shape with small changes per pass, there are ambiguities in the selection of the model parameters, such as the pulse duration. The resulting uncertainties are small compared to the drastic differences in the two regimes of operation, so useful comparisons can be made. With the parameters of laser configuration B (Table 1) pulse energies of ~ 220 nJ correspond to peak powers in the range of those obtained from the simulations and about ~ 35 fs chirped pulses. However, the

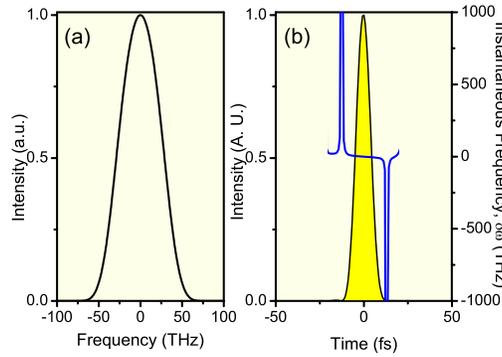


Fig. 2. Simulated output pulse generated with net anomalous GVD, corresponding to laser configuration A. (a) Power spectra, (b) intensity profile and instantaneous frequency.

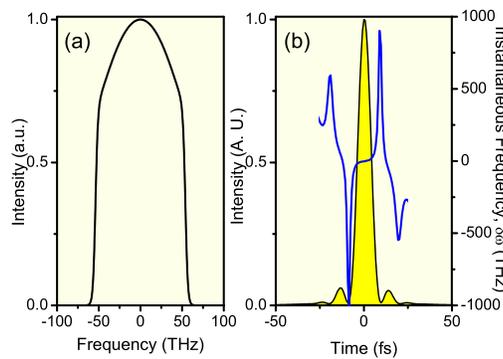


Fig. 3. Simulated relatively low-energy output pulse generated with net normal GVD of 15 fs^2 , corresponding to laser configuration B. (a) Power spectra, (b) intensity profile and instantaneous frequency after dechirping.

model predicts the operation to be highly unstable. The saturable absorption can be increased to stabilize the pulses, but then the peak power would be ~ 10 times higher, leading to instabilities. In either case, one would conclude that a stable pulse train would never be observed based on the analytic model.

Much higher pulse energies are achievable at higher values of total GVD (laser configuration C). With the dispersion fixed at $+30 \text{ fs}^2$, the stable pulse energy can be increased by a factor of ~ 20 , while retaining a similar peak power within the Ti:sapphire crystal (Fig. 5). The increased energy is accommodated despite a decrease in the stretching ratio compared to laser B. The pulse energy would need to be somewhat higher to maintain the stretching ratio, which would also enhance the linearity of the chirp; however, we limit the pulse energy to avoid excessive peak power within the crystal. Alternatively, if the magnitude of GVD can be increased, the stretching (and consequently the pulse quality) will increase. However, these are details; 20 times larger pulse energy is obtained, in exchange for small degradation in the pulse quality and time-bandwidth product. Simulations indicate a continuing trend of increasing pulse energy with increasing positive GVD, while maintaining the peak power. For instance, with the cavity dispersion set to $+45 \text{ fs}^2$, intra-cavity pulse energy of $2.3 \mu\text{J}$ results in 8 MW peak power within the Ti:sapphire crystal. This trend continues to arbitrarily high energies within

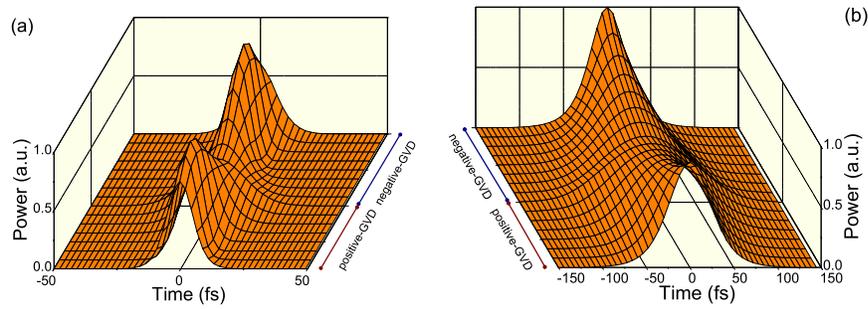


Fig. 4. Pulse shaping in one round trip: (a) DM-soliton mode (laser configuration A), (b) self-similar evolution mode (laser configuration B).

this simplified model neglecting spatial effects. Within this model, the physics remains qualitatively unchanged as long as the total GVD is small in comparison to that of the normal and anomalous GVD segments.

All of the simulations assume 10% output coupling, which would correspond to 100 nJ pulse energy for laser configuration C. Low repetition rate, high-energy Ti:sapphire lasers demonstrated to date have employed output coupling ratios as high as 25% [11]. Here we assumed $\gamma = 1 \text{ (MW cm)}^{-1}$, which can easily be decreased by 4 times through the use of a cavity mode larger by a factor of 2. Therefore, with proper design, extraction of 1 μJ or higher pulse energies looks feasible with this approach. The practical demonstration of such performance may require a lower repetition rate and greater pump power than has been used to date, in order to store sufficient energy within the cavity. Utilizing recently-available pump lasers with 18 W of average power, and scaling from the results of [11] we anticipate that a 3-4 MHz repetition-rate cavity should be able to generate pulses with 1 μJ energy.

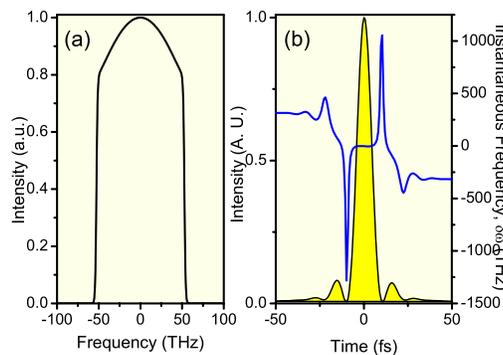


Fig. 5. Simulated output pulse generated with net normal GVD and high pulse energy, corresponding to laser configuration C. (a) Power spectra, (b) intensity profile and instantaneous frequency after dechirping.

In conclusion, numerical simulations indicate that it should be possible to operate solid-state lasers under conditions that approximate the self-similar regime, which so far has been explored only in fiber lasers. The main benefit of the self-similar regime applied to solid-state lasers is that much larger energies can be stored in highly-chirped pulses, which maintain acceptable peak powers inside the cavity, while retaining a broad spectrum with a linear chirp. The benefit

of large bandwidth and linearity of the chirp is in contrast to previously-observed positive-GVD operation of Ti:sapphire lasers with negligible pulse breathing. The small dispersion of solid-state gain media presents a significant challenge, and will likely prevent the benefits of self-similar propagation from being realized as thoroughly as they have been in fiber lasers. On the other hand, the large gain bandwidth of Ti:sapphire (and other gain media such as Cr:LiSAF, e.g.) and the availability of a fast and monotonically-saturating absorber (the Kerr lens) simplify the design in comparison to fiber lasers. Even the marginal achievement of the self-similar regime can produce order-of-magnitude increases in pulse energy, at the expense of minor decreases in pulse quality. The goal of the present study was simply to assess the feasibility of achieving self-similar operation of a solid-state laser. One could certainly consider the insertion of material with normal GVD and positive nonlinearity (glass, e.g.) in the cavity, to better decouple gain filtering from the self-similar evolution. If the operation can be optimized to obtain spectral bandwidths twice as large as those shown here, more significant increases in pulse energy will follow. Ultimately, energies of at least $1 \mu\text{J}$ should be generated in pulses ~ 10 fs in duration.

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